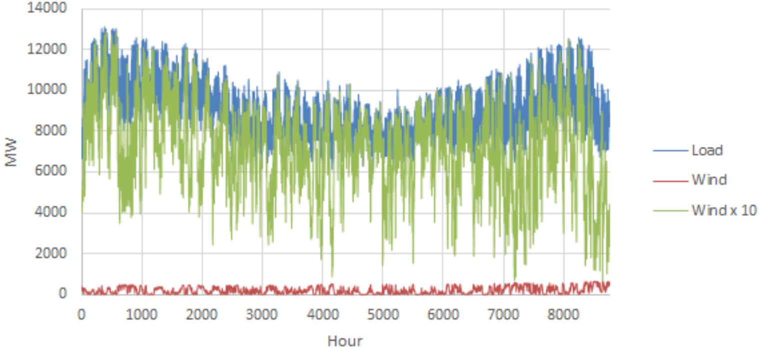
Capacity Expansion

Operations Research

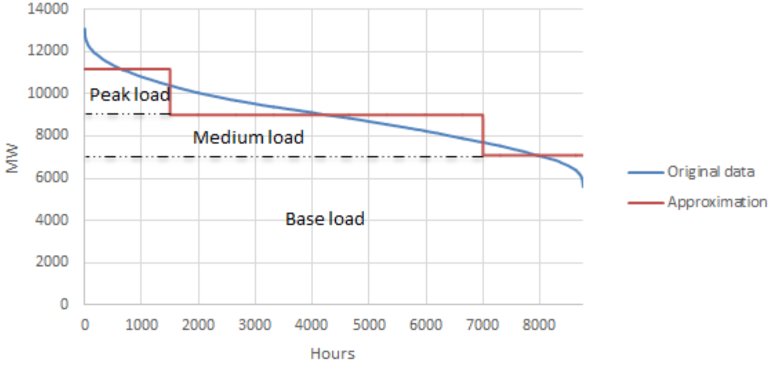
Anthony Papavasiliou

Outline

Load and Wind in Belgium, 2013



Load Duration Curve



Load duration curve is obtained by sorting load time series in descending order

Horizontal Stratification of Load

Load duration curve describes number of hours in the year that load was greater than or equal to a given level (e.g. net load was $\geq$ 10000 MW for 2000 hours)

Step-wise approximation:

- Base load: 0-7086 MW, lasts for 8760 hours (whole year)
- Medium load: 7086-9004 MW, lasts for 7500 hours
- Peak load: 9004-11169 MW, lasts for 1500 hours

Technological Options

Technology	Fuel cost ($/MWh)	Inv cost ($/MWh)
Coal	25	16
Gas	80	5
Nuclear	6.5	32
Oil	160	2
DR	1000	0

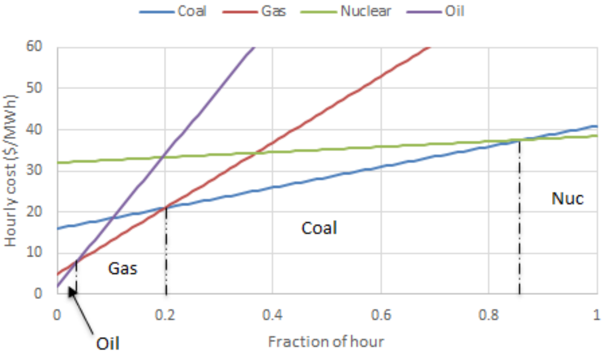
- Fuel/variable cost: proportional to energy produced
- Investment/fixed cost: proportional to built capacity
- Discounted investment cost: *hourly* cash flow required for 1 MW of investment

Optimal Investment Problem

Optimal investment problem: find mix of technologies that can serve demand at minimum total (fixed + variable) cost

The optimal investment problem can be solved graphically with *screening curves*

Screening Curves

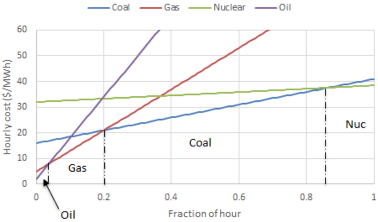


Screening curve: Total hourly cost as a function of the fraction of time that a technology is producing

Logic of Graphical Solution

- Total cost of using 1 MW of a technology depends on amount of time it produces
- Each *horizontal slice* of load can be allocated to an optimal technology, depending on its duration (which technology should serve base load? peak load?)

Optimal Solution



Fraction of time each technology should be functioning:

- DR: $1000 \cdot f \leq 2 + 160 \cdot f \Leftrightarrow f \leq 0.0024 \Rightarrow$ 0-21 hours
- Oil: $f > 0.0024$ and $2 + 160 \cdot f \leq 5 + 80 \cdot f \Leftrightarrow f \leq 0.0375 \Rightarrow$ 21-328 hours
- Gas: $f > 0.0375$ and $5 + 80 \cdot f \leq 16 + 25 \cdot f \Leftrightarrow f \leq 0.2 \Rightarrow$ 328-1752
- Coal: $f > 0.2$ and $16 + 25 \cdot f \leq 32 + 6.5 \cdot f \Leftrightarrow f \leq 0.8649 \Rightarrow$ 1752-7576 hours
- For nuclear: $0.8649 \leq f \leq 1 \Rightarrow$ 7576-8760 hours

Optimal Solution

Recall,

- Base load: 0-7086 MW, lasts for 8760 hours (whole year)
- Medium load: 7086-9004 MW, lasts for 7500 hours
- Peak load: 9004-11169 MW, lasts for 1500 hours

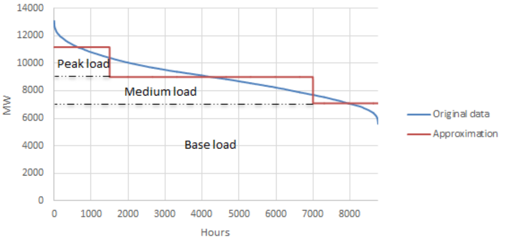
From previous slide,

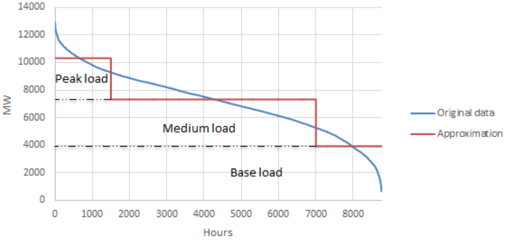
- Base-load is assigned to nuclear: 7086 MW
- Medium load is assigned to coal: 1918 MW
- Peak load is assigned to gas: 2165 MW
- No load is assigned to oil: 0 MW
- No load is assigned to DR: 0 MW

Outline

Increasing Wind Penetration

Which load duration curve corresponds to 10x wind power?





Scenarios

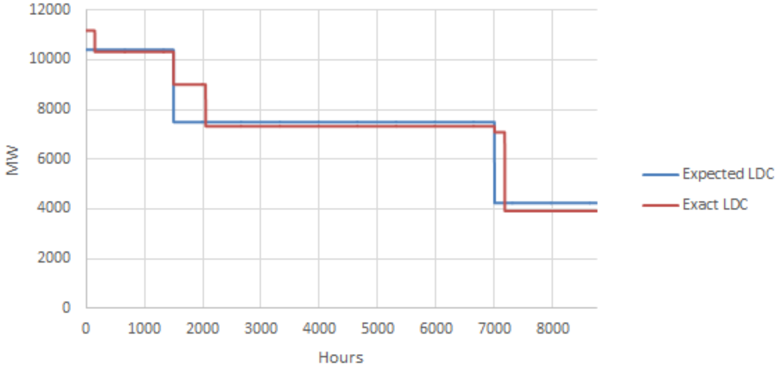
	Duration (hours)	Level (MW) Ref	Level (MW) 10x wind
Base load	8760	0-7086	0-3919
Medium load	7000	7086-9004	3919-7329
Peak load	1500	9004-11169	7329-10315

- Ref wind: 10%
- 10x wind: 90%

Goal determine *optimal* expansion plan

Optimal refers here to the expansion plan that minimizes the *expected* total cost.

Stochastic Program Vs Expected Value Problem



How do we compute each load duration curve?

Screening Curve Solution

	Duration (hours)	Level (MW)	Technology
Block 1	8760	0-3919	Nuclear
Block 2	7176	3919-7086	Coal
Block 3	7000	7086-7329	Coal
Block 4	2050	7329-9004	Coal
Block 5	1500	9004-10315	Gas
Block 6	150	10315-11169	Oil

Table: Optimal assignment of capacity for the 6-block load duration curve.

	Duration (hours)	Level (MW)	Technology
Base load	8760	0-4235	Nuclear
Medium load	7000	4235-7496	Coal
Peak load	1500	7496-10401	Gas

Table: Optimal assignment of capacity for the expected load duration curve.

Investment and Fixed Cost

	SP inv (MW)	EV inv (MW)	SP fixed cost ($/h)	EV fixed cost ($/h)
Coal	5085	3261	81360	52176
Gas	1311	2905	6555	14525
Nuclear	3919	4235	125408	135520
Oil	854	0	1708	0
Total	11169	10401	215031	202221

- Why are the investment plans different?
- Why does the EV solution have a lower fixed cost?

Merit Order Dispatch

Merit order dispatch rule: In order of increasing variable cost, assigns technologies to load blocks of decreasing duration, until either all load blocks are satisfied or all generating capacity is exhausted

Variable Cost

	SP var cost ($/h)	EV var cost ($/h)
Block 1	25473	25473
Block 2	64858	60070
Block 3	4854	4854
Block 4	9799	29209
Block 5	17960	17959
Block 6	2340	13268
Total	125285	150834

The EV solution is expensive in serving block 4 (served largely by gas instead of coal) and block 6 (why?)

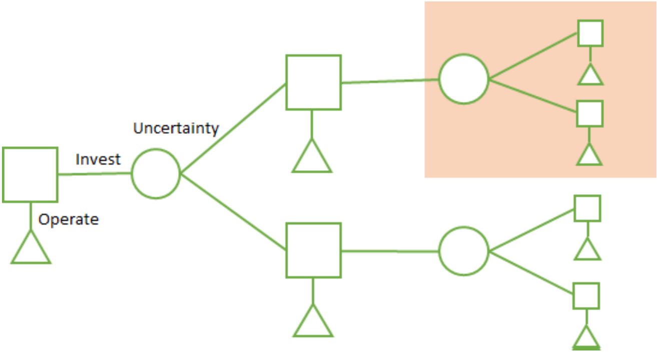
Value of the Stochastic Solution

Value of the stochastic solution (VSS): Cost difference of stochastic programming solution and expected value solution when the two are compared against the '*true*' model of uncertainty

- Stochastic program: 125285 (variable) + 215031 (fixed) = 340316 $/h
- Expected value problem: 150834 (variable) + 202221 (fixed) = 353055 $/h

$$VSS = 12739 \text{ \$/h}$$

Multiple Periods



- Orange area: sub-structure that recurs as we move backwards $\Rightarrow$ **dynamic programming**
- **Block separability**: some decisions do not influence the future state of the system, only the payoff of each period (which one matters for the future, 'Invest' or 'Operate'?)

Math Programming Formulation of 2-Stage Problem

$$\min_{x,y \geq 0} \sum_{i=1}^{n} (I_i \cdot x_i + \sum_{j=1}^{m} C_i \cdot T_j \cdot y_{ij})$$

$$\text{s.t.} \sum_{i=1}^{n} y_{ij} = D_j, j = 1, \ldots, m$$

$$\sum_{j=1}^{m} y_{ij} \leq x_i, i = 1, \ldots n - 1$$

- I_i, C_i: fixed/variable cost of technology i
- D_j, T_j: height/width of load block j
- y_{ij}: capacity of i allocated to j
- x_i: capacity of i

Where is the uncertainty?

Towards a Dynamic Programming Algorithm

In order to solve multi-stage problem via dynamic programming, we would like to express cost of 2-stage problem as a function of investment x

Consider the following LP, with fixed x:

$$
\begin{aligned}
f(x) \quad = \quad & \min_{y \geq 0} \sum_{i=1}^{n} (I_i \cdot x_i + \sum_{j=1}^{m} C_i \cdot T_j \cdot y_{ij}) \\
& \text{s.t. } \sum_{i=1}^{n} y_{ij} = D_j, j = 1, \ldots, m \\
& \sum_{j=1}^{m} y_{ij} \leq x_i, i = 1, \ldots, n-1
\end{aligned}
$$

Show that $f(x)$ is a piecewise linear function of x