# Demand Response

## Quantitative Energy Economics

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# Demand Response

**Demand response**: active participation of consumers in (i) efficient consumption of electricity and (ii) provision of ancillary services

Types of demand response:

1. Efficiency
2. Peak load shaving
3. Load shifting

# Retail Pricing

Mechanisms for retail pricing of electricity:

- Real-time pricing
- *Time of use pricing*
- Critical peak pricing: ToU + critical peak events
- *Interruptible service*

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# Motivation of Time of Use Pricing

- Electricity service consists of (i) fuel cost for producing power, and (ii) investment cost for building capacity
- If electricity were priced at marginal fuel cost, demand in peak periods would be too high
- ToU pricing breaks bill into two parts:
  1. energy component: charge proportional to amount of power consumption, differs depending on the time of day
  2. capacity component: applied to consumers who contribute to need of installing additional capacity to the system
- Goal is to flatten demand across time periods

# Simple Two-Period Model

Consider the following system:

- Decreasing marginal benefit functions:
  - Peak: $MB_1(p)$, lasts fraction $\tau_1$
  - Off-peak: $MB_2(p)$, lasts fraction $\tau_2$
- Increasing marginal investment cost $MI(x)$, with $MI(x) > 0$ for all $x$
- Increasing marginal fuel cost $MC(p)$
- Suppose $MB_1(0) > MC(0) + \frac{MI(0)}{\tau_1}$

# Welfare Maximization Model

Denote

- $x$: amount of constructed capacity
- $p_1$ [$p_2$]: production in peak [off-peak] hours

$$
\begin{aligned}
& \max \tau_1 \int_0^{p_1} MB_1(q)dq + \tau_2 \int_0^{p_2} MB_2(q)dq \\
& - \int_0^{x} MI(q)dq - \tau_1 \int_0^{p_1} MC(q)dq - \tau_2 \int_0^{p_2} MC(q)dq \\
(\rho_1 \tau_1): \quad & p_1 \leq x \\
(\rho_2 \tau_2): \quad & p_2 \leq x \\
& p_1, p_2, x \geq 0
\end{aligned}
$$

Note: since $MI(x) > 0$, in the optimal solution $p_1 = x$, $p_2 = x$, or both

# KKT Conditions

$$0 \leq \rho_1 \perp x - p_1 \geq 0$$
$$0 \leq \rho_2 \perp x - p_2 \geq 0$$
$$0 \leq p_1 \perp -MB_1(p_1) + MC(p_1) + \rho_1 \geq 0$$
$$0 \leq p_2 \perp -MB_2(p_2) + MC(p_2) + \rho_2 \geq 0$$
$$0 \leq x \perp MI(x) - \rho_1\tau_1 - \rho_2\tau_2 \geq 0$$

Note: dual multipliers have been scaled by $\tau_i$

# Marginal Cost Pricing Is Sub-Optimal

**Proposition**: Suppose that electricity is priced at the marginal variable cost $MC(p_i)$ for each period $i$. This will result in suboptimal investment if the system is built so as to make sure that no demand can be left unserved.

Mathematically: Optimal solution cannot satisfy all of the following conditions:

- $MC(p_1) = MB_1(p_1)$
- $MC(p_2) = MB_2(p_2)$
- $x = \max(p_1, p_2)$

Proof: By contradiction, using KKT conditions

We first show $\rho_1 = \rho_2 = 0$:

- Since $MB_1(0) > MC(0) + \frac{MI(0)}{\tau_1}$, optimal investment must be such that $x > 0$
- Suppose $\rho_i > 0$, then $p_i = x > 0$
- Since $p_i > 0$, $MB_i(p_i) = MC(p_i) + \rho_i > MC(p_i)$
- Marginal cost pricing requires $MB_i(p_i) = MC(p_i)$, hence $\rho_1 = \rho_2 = 0$

We then show $\rho_i > 0$ for some $i$:

- Since $x > 0$, by complementarity

$$MI(x) = \rho_1 + \rho_2$$

- Since $MI(x) > 0$ for all $x$, $\rho_i > 0$ for $i = 1$, or $i = 2$, or both

# Peak Charges

Interpretation of multiplier $\rho_i$: charge above the marginal cost of the marginal technology, $MC(p_i)$

For constant marginal investment cost, $MI(x) = MI$, additional charges are exactly equal to capital investment costs

# Example: Pricing Peak and Off-Peak

Consider the following market:

- $MI(x) = 5$ \$/MWh
- $MC(p) = 80$ \$/MWh
- Peak demand $MB_1(p) = \max(1000 - p, 0)$ \$/MWh, with $\tau_1 = 20\%$
- Off-peak demand $MB_2(p) = \max(500 - p, 0)$ \$/MWh, with $\tau_2 = 80\%$

**Problem**: You are told that optimal investment is $x = 895$ MW. What are the optimal ToU prices?

- Since optimal $x$ is 895 MW, then either $p_1 = 895$ MW, $p_2 = 895$ MW, or both
- Check that $MB_1(895) = 105$ \$MWh and $MB_2(895) = 0$ \$/MWh
- Obviously $p_2 < x$ (marginal benefit at 895 MW is zero, marginal cost is 80 \$/MWh)
- Therefore, $p_1 = 895$ MW
- Price in peak periods: 105 \$/MWh
- From KKT conditions,

$$MB_2(p_2) = MC(p_2)$$

- Price in off-peak periods: 80 \$/MWh

# Graphical Illustration of Tariff

Consider the fixed retail tarrif which is average of ToU tariff:

$$0.2 \cdot 105 + 0.8 \cdot 80 = 85\$/\text{MWh}$$

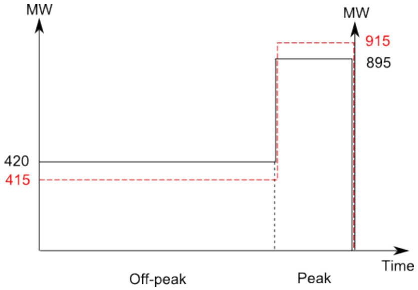


Figure: Demand under fixed retail pricing (black solid curve) and time of use pricing (red dashed curve). Effect of ToU pricing: depresses consumption in peak hours, increases consumption in off-peak hours.

# Example: Sharing Peak Charges

Consider the previous example, with $MB_2(p_2) = 980 - p$ \$/MWh (and everything else unchanged)

Price of 80 \$/MWh in off-peak hours violates installed capacity

Optimal solution: $x = 899$ MW, $p_1 = p_2 = 899$ MW

Sharing of capital costs among peak and off-peak consumers:

- $\rho_1/\tau_1 = 21$ \$/MWh
- $\rho_2/\tau_2 = 1$ \$/MWh

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# System Reliability

Define

$$r(v) = F(D(v))$$

where

- $D(v)$: demand function (power demand resulting from consumers who value power at $v$ *or more*)
- $F(L)$: probability of having $L$ MW *or more* of power available

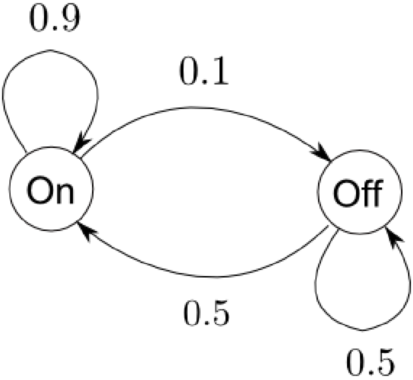
Interpretation of $r(v)$: probability of being able to satisfy consumers with valuation $v$ or higher

# Example

Consider the following system:

- Reliable technology: 295 MW
- Unreliable technology: 1880 MW
- Demand function: $D(v) = 1620 - 4v$

Unreliable technology described by Markov chain




Stationary distribution: $\pi_{\text{off}} = 0.167$, $\pi_{\text{on}} = 0.833$

- Generator availability:

$$F(L) = \begin{cases} 1, & L \leq 295 \text{ MW} \\ 0.833, & 295 \text{ MW} < L \leq 2175 \text{ MW} \\ 0, & L > 2175 \text{ MW} \end{cases}$$

- Service reliability:

$$r(D(v)) = \begin{cases} 0.833, & 0 \text{ \$/MWh} \leq v \leq 331.25 \text{ \$/MWh} \\ 1, & 331.25 \text{ \$/MWh} < v \leq 405 \text{ \$/MWh} \end{cases}$$

# Priority Service Contracts

**Priority service contracts** are defined as

$$(r, p(r))$$

where $r$ is the reliability of service and $p(r)$ is the price paid for $r$

Note: $p(r)$ will determine reliability chosen by customers

Goal: design $p(r)$ so that customers with higher valuation receive more reliable service

# Steering Customer Choice

Load with valuation $v$ selects reliability by solving

$$\max_r r \cdot v - p(r)$$

First order condition:

$$v - p'(r) = 0$$

Suppose $p(r)$ satisfies:

$$\begin{aligned} p'(r(D(v))) &= v && (1) \\ r \cdot v - p(r) &\geq 0 && (2) \end{aligned}$$

Load with valuation $v$

- is willing to procure a reliability contract
- chooses reliability level $r(D(v))$

# Computing the Price Menu

Integrating equation (1) by parts:

$$\hat{p}(v) = p_0 + \int_{v_0}^{v} y \cdot dr(D(y)) = v \cdot r(D(v)) - \int_{v_0}^{v} r(D(y))dy \quad (3)$$

where $v_0$ is **cutoff valuation**: valuation of cheapest customer who chooses to buy a priority service contract

Parametrizing with respect to $v$, the menu $(r, p(r))$ is

$$\{r(D(v)), \hat{p}(v), v \in [v_0, V]\}$$

where $V$ is maximum valuation

# Fixed Charge

Fixed charge $p_0$ determines cutoff valuation $v_0$:

$$v_0 \cdot r(v_0) - p_0 = 0 \tag{4}$$

Customers with $v < v_0$ do not procure reliability contracts

# Example Continued

$$r(v) = \begin{cases} 0.833, & 0 \le v \le 331.25 \\ 1, & 331.25 < v \le 405 \end{cases}$$

Suppose $v_0 = 10$ $/MWh, then from equation (4):

$$p_0 = 10 \cdot 0.833 = 8.33 \text{ \$/MWh}$$

From equation (3):

$$\begin{aligned} \hat{p}(v) &= p_0 + \int_{v_0}^{v} u \cdot dr(u) \\ &= \begin{cases} 8.33, & 10 \le v \le 331.25 \\ 8.33 + 331.25 \cdot 0.167, & 331.25 < v \le 405 \end{cases} \\ &= \begin{cases} 8.33, & 10 \le v \le 331.25 \\ 63.65, & 331.25 < v \le 405 \end{cases} \end{aligned}$$

Parametrizing with respect to $v$:

$$p(r) = \begin{cases} 8.33, & r = 0.833 \\ 63.65, & r = 1 \end{cases}$$

This is a menu with 2 options

# Load Self-Selection

Consider choice of load with valuation $v$:

$$\max\{0, 0.833 \cdot v - 8.33, v - 63.65\}$$

- $r = 0$ is optimal if $0.833 \cdot v - 8.33 \leq 0$ and $v - 63.65 \leq 0$, i.e. $v \leq 10$.
- $r = 0.833$ is optimal if $0 \leq 0.833 \cdot v - 8.33$ and $v - 63.65 \leq 0.833 \cdot v - 8.33$, i.e. $10 \leq v \leq 331.25$.
- $r = 1$ is optimal if $0 \leq v - 63.65$ and $0.833 \cdot v - 8.33 \leq v - 63.65$, i.e. $v \geq 331.25$.

# Different Choice of Fixed Charge

If menu designer would like all customers to procure reliability contracts, i.e. $v_0 = 0$, then $p_0 = 0$ and

$$p(r) = \begin{cases} 0, & r \leq 0.833 \\ 55.32, & 0.833 < r \leq 1 \end{cases}$$

# Service Policy

In case of shortage, customers with higher $r$ served first

*Note:* In order to design the menu, we used *aggregate* information ($r(L)$ and $D(v)$)

Menu selections allow us to dispatch *individual* customers efficiently!