Unit Commitment

Quantitative Energy Economics

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Unit Commitment

1. Day-Ahead and Real-Time Operations

2. Optimization Models of Unit Commitment
   - Security Constrained Unit Commitment

3. Market Design for Unit Commitment

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Day-Ahead and Real-Time Opertions

Day-ahead operations

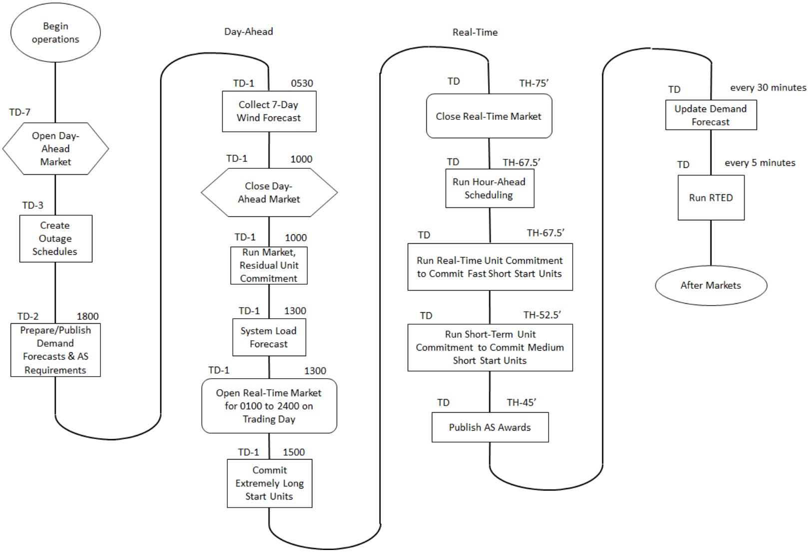
- Performed 24-36 hours in advance
- Necessary because of delays in starting / moving units
- Based on forecasts (of demand, renewable energy , system state)
- Unit commitment

Real-time operations

- Continuously
- Economic dispatch

Distinction between day-ahead scheduling and real-time dispatch is universal across systems

Flow Chart of Operations



The Real Thing

pjm

Day-ahead Market – Average Daily Volumes

- 1,210 generators, 3 part offers (startup, no load, 10 segment incremental energy offer curve)
- 10,000 - Demand bids – fixed or price sensitive
- 50,000 - Virtual bids / offers
- 8,700 - eligible bid/offer nodes (pricing nodes)
- 6,125 - monitored transmission elements
- 10,000 - transmission contingencies modeled

Computational Methods

Unit commitment is a large-scale mixed integer linear program

- Until 1960s: dispatch in order of increasing marginal cost
- 1970s, 1980s: dynamic programming with Lagrangian relaxation
- Past decade: branch and bound solvers

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Total Cost

Denote

- $VC_g(p_{gt})$: variable cost
- $FC_g(u_g)$: fixed cost
- $TC_g(u_g)$: total cost

$$TC_g(u_g, p_g) = FC_g(u_g) + \sum_{t=1}^{T} VC_g(p_{gt})$$

- T: scheduling horizon
- u_{gt}: indicate whether unit is on or off, with $u_g = (u_{g1}, \ldots, u_{gT}) \in \{0, 1\}^T$
- p_{gt}: power production, with $p_g = (p_{g1}, \ldots, p_{gT}) \in \mathbb{R}^T$
- r_{gt}: reserve, with $r_g = (r_{g1}, \ldots, r_{gT}) \in \mathbb{R}^T$

Example

Denote

- S_g: startup cost
- K_g: minimum load cost
- $MC_g(\cdot)$: marginal cost function

$$TC_g(u_g, p_g) = \sum_{t=1}^{T}(K_g u_{gt} + S_g v_{gt} + \int_0^{p_{gt}} MC_g(x)dx)$$

v_{gt}: indicator for startup in period t

$$v_{gt} = \begin{cases} 1 & \text{if } u_{g,t-1} = 0, u_{gt} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Generic Unit Commitment Model

$$
\begin{aligned}
(UC): \quad & \min \sum_{g \in G} TC_g(u_g, p_g) \\
& h_g(p_g, r_g, u_g) \leq 0 \\
& \sum_g p_{gt} = D_t \\
& \sum_g r_{gt} = R_t
\end{aligned}
$$

- h_g: private operating constraints of unit g
- D_t: power demand
- R_t : reserve demand

Initial Conditions

Denote

- $u0_g \in \{0,1\}^{T_0}$: initial commitment, T_0 periods prior to first period of scheduling horizon
- $p0_g \in \mathbb{R}^{T_0}$: initial production

How long should T_0 be?

Transitions

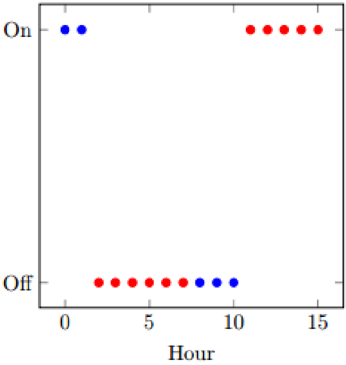
Notation:

- u indicates on status
- v indicates startup
- z indicates shutdown

$$u_{gt} = u_{g,t-1} + v_{gt} - z_{gt}.$$

Min Up/Down Times

- Red marks: forced states
- Blue marks: free choices



What is the min up time? down time?

Denote

- UT_g: min up time
- DT_g: min down time

$$\sum_{\tau=t-UT_g+1}^{t} v_{g\tau} \leq u_{gt}, t \geq UT_g$$

$$\sum_{\tau=t-DT_g+1}^{t} z_{g\tau} \leq 1 - u_{gt}, t \geq DT_g$$

Generator Temperature

Temperature of a generator determines how much fuel is required in order to start it up

Example:

- Hot: 200 GJ needed to start 1-16 hours after shut down
- Warm: 220 GJ needed to start 17-24 hours after shut down
- Cold: 250 GJ needed to start 25+ hours after shut down

$\Theta = \{\text{Hot, Warm, Cold}\}$

Temperature Dependent Startup

v_{glt}: indicator for startup in temperature state l at period t

Generator can only start up from a single temperature state:

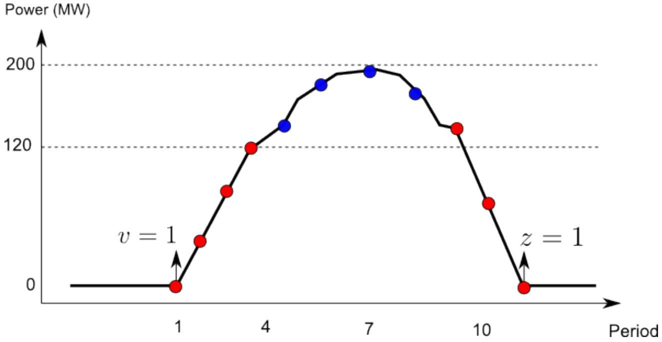
$$v_{gt} = \sum_{l \in \Theta} v_{glt}$$

Temperature state l occurs within $\underline{T}_{gl}$ to $\bar{T}_{gl}$ periods after shutdown:

$$v_{glt} \leq \sum_{\tau = t - \underline{T}_{gl} + 1}^{t - \bar{T}_{gl}} z_{g\tau}, \; t \geq \underline{T}_{gl}$$

Startup/Shutdown Profiles

Startup/shutdown profiles: predefined sequence of production when generators are started up / shut down



- Red points: startup profile (restricted)
- Blue circles: free dispatch

Example

Consider a generator with

- technical minimum: 120 MW (should be reached as soon as possible)
- ramp rate: 1 MW/min

Startup profile is (60 MW, 120 MW), why?

Temperature Dependent Startup Profiles

- u_{gt}^{SU}: indicator for startup
- u_{gt}^{SD}: indicator for shutdown
- u_{gt}^{DISP}: indicator for free dispatch

Generator must be in one of three states:

$$u_{gt} = u_{gt}^{SU} + u_{gt}^{DISP} + u_{gt}^{SD}$$

- T_{gl}^{SU}: duration of startup profile (depends on temperature l)
- T_{g}^{SD}: duration of shutdown profile

Determine whether generator is in startup/shutdown:

$$u_{gt}^{SU} = \sum_{l \in \Theta} \sum_{\tau = t - T_{gl}^{SU} + 1}^{t} v_{gl\tau}, t \geq \max_{l \in \Theta} T_{gl}^{SU}$$

$$u_{gt}^{SD} = \sum_{\tau = t}^{t + T_{g}^{SD} - 1} z_{g\tau}, t \leq T - T_{g}^{SD} + 1$$

Startup/Shutdown Production

- $P^{SU}_{gl\tau}$: sequence of production levels for startup profile (note dependence on temperature l)
- $P^{SD}_{g\tau}$: sequence of production levels for shutdown profile

Production in startup/shutdown profile:

$$p^{SU}_{gt} = \sum_{l \in \Theta} \sum_{\tau = t - T^{SU}_{gl} + 1}^{t} v_{gl\tau} P^{SU}_{gl, t - \tau + 1}, t \geq \max_{l \in \Theta} T^{SU}_{gl}$$

$$p^{SD}_{gt} = \sum_{\tau = t+1}^{t + T^{SD}_{gl}} z_{g\tau} P^{SD}_{g, \tau - t}, t \leq T - T^{SD}$$

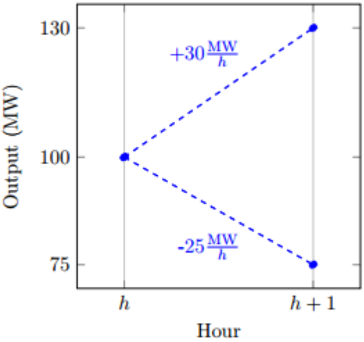
Dispatchable Production

Denote P_g^-, P_g^+ as technical minimum/maximum

$$p_{gt} \geq p_{gt}^{SU} + p_{gt}^{SD} + P_g^- u_{gt}^{DISP}$$
$$p_{gt} \leq p_{gt}^{SU} + p_{gt}^{SD} + P_g^+ u_{gt}^{DISP}$$

What happens when $u_{gt}^{DISP} = 0$? $u_{gt}^{DISP} = 1$?

Ramp Rates



Note: ramp rates may be violated by startup/shutdown profiles

Denote R_g^+, R_g^- as ramp up/down rate limit

$$\begin{aligned} p_{gt} - p_{g,t-1} &\leq R_g^+ + M u_{gt}^{SU}, t \geq 2 \\ p_{g,t-1} - p_{gt} &\leq R_g^- + M u_{gt}^{SD}, t \geq 2 \end{aligned}$$

What happens when $u_{gt}^{DISP} = 0$? $u_{gt}^{DISP} = 1$?

Fixed Costs

Denote

- SUC_{gl}: startup cost for temperature l
- MLC_g: minimum load cost

$$FC(u_g) = \sum_{t=1}^{T}(\sum_{l \in \Theta} SUC_{gl} v_{glt} + MLC_g u_{gt})$$

Note: Fuel cost from startup profiles *not* accounted here

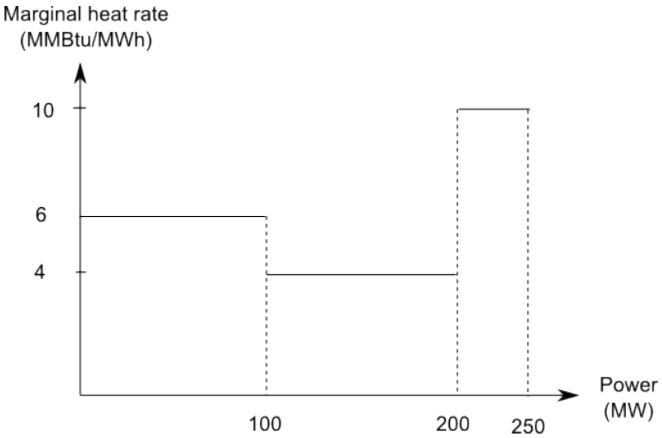
Variable Costs

- **Average heat rate** (MMBtu/MWh): ratio of *total* fuel consumption to *total* electric power production
- **Marginal heat rate** (MMBtu/MWh): derivative of fuel consumption with respect to electric power production

Denote $MHR_g(p)$ as marginal heat rate curve, FP as fuel price ($/MMBtu):

$$VC(p_{gt}) = FP \int_0^{p_{gt}} MHR(x)dx.$$

Non-Increasing Marginal Heat Rate



Why does this heat rate curve cause modeling problems?

Denote:

- S: set of segments in heat rate curve
- P^+_{gs}: width of each segment
- MHR_{gs}: marginal heat rate of each segment

Activate first segment once generator is started up:

$$u_{gs_1 t} = u_{gt}$$

Segment cannot be activated before previous segment is fully used:

$$u_{g,s+1,t} \leq \frac{p_{gst}}{P^+_{gs}}$$

Production within each segment:

$$0 \leq p_{gst} \leq P^{+}_{gs} u_{gst}$$

Total power production:

$$p_{gt} = \sum_{s \in S} p_{gst}$$

Total variable cost:

$$VC_g(p_{gt}) = FP \sum_{s \in S} MHR_{gs} p_{gst}$$

Secondary Reserves

Denote upwards/downwards reserve as $r2^+_{gt}/r2^-_{gt} \geq 0$

Min/max capacity constraints:

$$p_{gt} - r2^-_{gt} \geq p^{SU}_{gt} + p^{SD}_{gt} + P^-_g u^{DISP}_{gt}$$
$$p_{gt} + r2^+_{gt} \leq p^{SU}_{gt} + p^{SD}_{gt} + P^+_g u^{DISP}_{gt}$$

Denote upward/downward reserve limits as $MR2^+_g/MR2^-_g$

$$r2^-_{gt} \leq MR^-_g u^{DISP}_{gt}$$
$$r2^+_{gt} \leq MR^+_g u^{DISP}_{gt}$$

Denote upward/downward requirements as $RR2^+_t/RR2^-_t$

$$\sum_{g \in G} r2^-_{gt} \geq RR2^-_t, \sum_{g \in G} r2^+_{gt} \geq RR2^+_t$$

Tertiary Reserves

Denote $r3^S_{gt} \geq 0$ as spinning reserve (on-line tertiary)

Max capacity:

$$p_{gt} + r2^+_{gt} + r3^S_{gt} \leq p^{SU}_{gt} + p^{SD}_{gt} + P^+_g u^{DISP}_{gt}$$

Denote $r3^{NS}_{gt} \geq 0$ as non-spinning reserve (off-line tertiary)

Max capacity:

$$r3^{NS}_{gt} \leq P^+_g(1 - u_{gt})$$

Denote $MR3_g$ as tertiary reserve limit:

$$r3^S_{gt} + r3^{NS}_{gt} \leq MR3_g$$

Denote aggregate reserve requirements as $RR3_t$:

$$\sum_{g \in G} (r3^S_{gt} + r3^{NS}_{gt}) \geq RR3_t$$

Reserve Requirements for Renewables

Unit commitment model can quantify

- Reserve requirements
- Operating cost
- Utilization of resources (conventional, renewable)

Policy support: we can quantify trade-offs of renewable energy

- Uncertainty (-)
- Free fuel cost (+)

The big question is: how many reserves do we need? Different models provide different answers...

Stochastic Unit Commitment

Two-stage formulation:

1. First stage: commitment
2. Revelation of uncertainty: component (generators, lines) failures, forecast errors (renewables, demand)
3. Second stage: generator/load dispatch

Setup

- Conventional units: controllable, costly
- Renewable generators: zero cost, unpredictable

Trade-off

- Too many reserves $\Rightarrow$ high startup/min load costs, renewable energy curtailment
- Too few reserves $\Rightarrow$ load shedding

Criticisms

- Model size
- Detailed model of uncertainty is needed
- Scenario selection is crucial and non-trivial

Security Constrained Unit Commitment

- Objective: minimize cost under normal conditions
- Each ‘scenario’ corresponds to the outage of a single component
- All demand must be satisfied
- Renewable supply replaced by forecast

- In line with approach of system operator to unit commitment (+)
- Large-scale problem (-)
- Conservative (-)

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Varieties of Day-Ahead Market Designs

Decentralized ⟵⟶ Centralized

Years ahead   Day ahead   Real time

⟶

We will analyze two variations:

- Exchanges (more decentralized)
- Pools (more centralized)

Role of Day-Ahead Markets

Day-ahead markets are *forward* markets for power

Two-settlement system: organization of (1) day-ahead markets as forward markets for trading power, followed by (2) a real-time market for settling imbalances

Two-Settlement System for Generators

Suppose generator *sells* Q_1 at P_1 in day-ahead market and *produces* Q_0 in real time:

- *Receive* $P_1 \cdot Q_1$ from day-ahead market
- If $Q_0 > Q_1$, *receive* P_0 for the extra power $Q_0 - Q_1$
- If $Q_0 < Q_1$, *pay* P_0 for the shortage $Q_1 - Q_0$

Generator is paid

$$R = P_1 \cdot Q_1 + P_0(Q_0 - Q_1)$$

Two-Settlement System for Loads

Suppose load *buys* Q_1 at P_1 in day-ahead market and *consumes* Q_0 in real time:

- *Pay* $P_1 \cdot Q_1$ from day-ahead market
- If $Q_0 > Q_1$, *pay* P_0 for the extra power $Q_0 - Q_1$
- If $Q_0 < Q_1$, *receive* P_0 for the leftover $Q_1 - Q_0$

Load pays

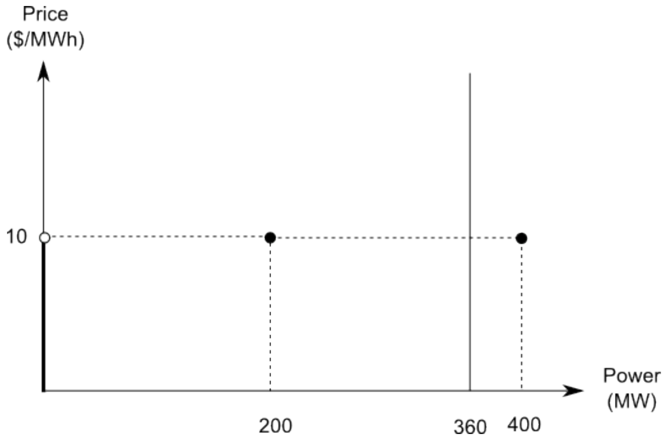
$$R = P_1 \cdot Q_1 + P_0(Q_0 - Q_1)$$

A System Without a Market-Clearing Price

Consider the following market:

- Inelastic demand: 360 MW
- Three identical generators
  - Capacity: 200 MW
  - Startup cost: 1000 $
  - Marginal cost: 5 $/MWh

Note: there is no price that exactly equilibrates supply and demand (why?)


Price
($/MWh)
10
200
360
400
Power
(MW)

Exchanges

Exchanges: uniform price auctions with simple bidding rules

- Bidders internalize fixed costs in their bids
- Less complicated rules (hence less gaming)
- More complicated strategy needed by generators (truthful bidding is suboptimal)

Example

Recall previous market:

- Inelastic demand: 360 MW
- Three identical generators
  - Capacity: 200 MW
  - Startup cost: 1000 $
  - Marginal cost: 5 $/MWh

- Bid below 10 $/MWh results in losses if in the money
- Bid at 10 $/MWh results in losses if in the money and generator produces 160 MW (instead of 200 MW)
- Pure strategy Nash equilibrium: bid at 11.25 $/MWh

What happened? Generators internalized fixed costs in bids

Pools

Pools are multi-part auctions where producers submit their costs and operating constraints, and different producers effectively receive different prices due to uplift payments

- Complex auction rules $\Rightarrow$ susceptible to gaming
- Simple for suppliers, complex for market operator
- Suppliers are paid differently because of uplift payments

Supplier bids. Suppliers submit *all* their information (fuel cost, startup cost, min load cost, ramp rates, min up/down times, etc)

Consumer bids. Consumers submit decreasing bids

Obligations and payoffs. Market operator solves (UC), and

1. determines a price for energy /reserves
2. suppliers/consumers obliged to follow (UC) solution
3. **Uplift payments**: payments from market operator to suppliers if their instructions are not profit-maximizing

Different market designs for pools, depending on rules for setting price

Setting Prices: Option 1 (O'Neill, 2001)

Get price as λ_t from following problem:

$$
\begin{aligned}
& \min \sum_{g \in G} TC_g(u_g, p_g) \\
& h_g(p_g, r_g, u_g) \leq 0 \\
(\lambda_t): \quad & \sum_g p_{gt} = D_t \\
& u_{gt} = u_{gt}^{\star}
\end{aligned}
$$

where $u_{gt}^{\star}$ is optimal solution of (UC)

Motivation: unit commitment provides 'price' information after fixing integer variables

Setting Prices: Option 2 (Hogan, 2003)

Get price λ_t from following problem:

$$\max_{\lambda} \phi(\lambda),$$

where

$$\begin{aligned} \phi(\lambda) \quad = \quad & \min_{p,r,u}\Big(\sum_{g\in G} TC_g(u_g, p_g) - \sum_{t} \lambda_t\big(\sum_{g\in G} p_{gt} - D_t\big)\Big) \\ & \text{s.t. } h_g(p_g, r_g, u_g) \leq 0 \end{aligned}$$

Motivation: find prices that minimize uplift payments of market operator

Example (Option 1, O'Neill)

Recall previous example, suppose suppliers bid truthfully:

- Inelastic demand: 360 MW
- Three identical generators
  - Capacity: 200 MW
  - Startup cost: 1000 $
  - Marginal cost: 5 $/MWh

Energy price determined from following problem (why?):

$$
\begin{aligned}
& \min 5p_1 + 5p_2 \\
(\lambda): \quad & p_1 + p_2 = 360 \\
& 0 \leq p_i \leq 200, i \in \{1, 2\}
\end{aligned}
$$

- Price: 5 $/MWh
- Uplift: 2000 $ (why?)

Example (Option 2, Hogan)

Dual function:

$$\begin{aligned} \phi(\lambda) &= \min_{p,u} 5p_1 + 5p_2 - \lambda(p_1 + p_2 - 360) \\ &\quad \text{s.t. } 0 \leq p_i \leq 200u_i \\ &\quad u_i \in \{0, 1\} \end{aligned}$$

Maximizer of $\phi(\cdot)$ equals 5 (why?)

- Price: 5 \$/MWh
- Uplift: 2000 \$
- Note: same price as option 1, in general not the case