Economic Dispatch

Quantitative Energy Economics

Anthony Papavasiliou

1. The Economic Dispatch Model
2. Competitive Market Equilibrium
3. Optimization Modeling of Market Equilibrium

What is Economic Dispatch?

- Simplest resource allocation problem in electricity markets
- Model used in *real-time* electricity markets
  - Uniform price auctions
  - Repeated every five to fifteen minutes

Outline

1. The Economic Dispatch Model
2. Competitive Market Equilibrium
3. Optimization Modeling of Market Equilibrium

Welfare Maximizing Economic Dispatch Problem

$$\max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$(\lambda): \quad \sum_{l \in L} d_l - \sum_{g \in G} p_g \leq 0$$

$$(\nu_l): \quad d_l \leq D_l, l \in L$$

$$(\mu_g): \quad p_g \leq P_g, g \in G$$

$$p_g \geq 0, g \in G$$

$$d_l \geq 0$$

- Set of loads L, set of generators G
- Increasing marginal cost $MC_g(\cdot)$
- Decreasing marginal benefit $MB_l(\cdot)$

KKT Conditions

$$0 \leq p_g \perp -\lambda + MC_g(p_g) + \mu_g \geq 0$$

$$0 \leq d_l \perp -MB_l(d_l) + \lambda + \nu_l \geq 0$$

$$0 \leq \mu_g \perp P_g - p_g \geq 0$$

$$0 \leq \nu_l \perp D_l - d_l \geq 0$$

$$0 \leq \lambda \perp \sum_{g \in G} p_g - \sum_{l \in L} d_l \geq 0$$

System Lambda

There exists a threshold λ such that:

1. If $0 < p_g < P_g$, then $MC_g(p_g) = \lambda$. If $0 < d_l < D_l$, then $MB_l(d_l) = \lambda$.
2. If $p_g = 0$, then $MC_g(0) \geq \lambda$. If $d_l = 0$, then $MB_l(0) \leq \lambda$.
3. If $p_g = P_g$, then $MC_g(P_g) \leq \lambda$. If $d_l = D_l$, then $MB_l(D_l) \geq \lambda$.

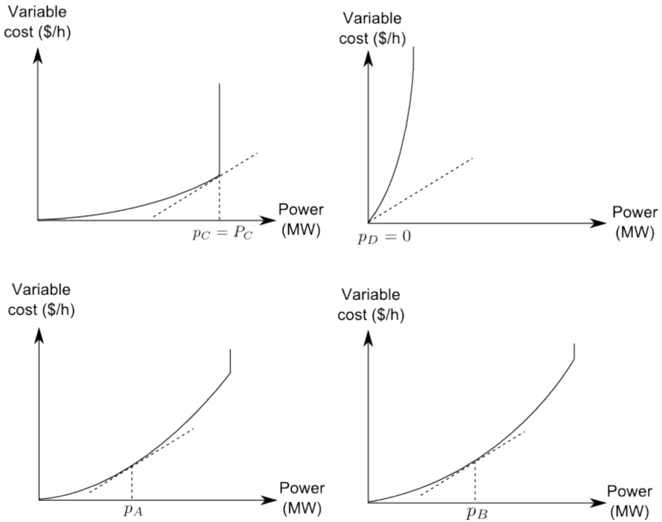
Proof: KKT conditions

System lambda: marginal cost of the marginal generating unit (i.e. the generating unit which will supply the next unit of power at lowest cost)

Interpretation of KKT Conditions

Optimal solution is matching cheapest generators with consumers who have greatest valuation (can you see why from the KKT conditions?)

Graphical Illustration of KKT Conditions



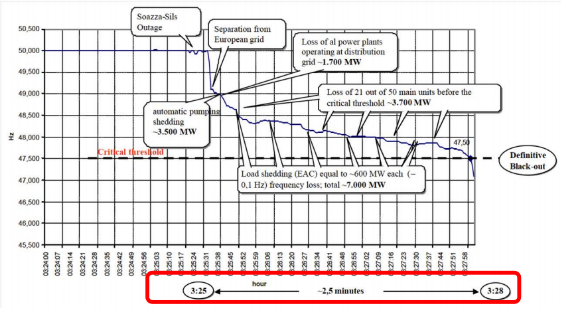
Outline

Path to Deregulation

1. Late 1970s: power systems are operated as vertically integrated regulated monopolies
2. Before 1980s: Premature markets (e.g. Norway)
3. 1982: Chile introduces a sport market
4. 1988: British government privatizes public power sector in England and Wales
5. 1990: Nordic market expands to include Sweden, Finland and Denmark
6. New Zealand and Australia introduced spot markets
7. The United States follow with California (CAISO), Pennsylvania-New Jersey-Maryland (PJM), Texas (ERCOT), New York (NYISO) and the Midwest (MISO)

Trading in Real Time

- Real-time markets cannot rely on bilateral negotiations (only takes a few minutes of imbalance for a blackout)
- ... but they can rely on a uniform price auction that charges system lambda for power
- But why is system lambda the 'right' price?



Definition of Competitive Market

A market is **competitive** if:

- Agents are price-taking
- Variable cost is convex and benefit is concave (which implies that marginal cost is? marginal benefit is?)
- Agents have access to public information (prices)

Aggregate and Marginal Cost

Aggregate cost is the cheapest way to produce Q MW of power among a *collection* of producers

$$TC_G(Q) = \min \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$\text{s.t.} \sum_{g \in G} p_g = Q$$

$$p_g \in \text{dom } MC_g$$

Marginal cost: $MC_G(Q) = TC'_G(Q)$

- Constraints imposed through domain of objective function
- What do we know about MC in competitive markets?
- What is the unit of measurement of TC? MC?

Merit Order Curve

Merit order curve = (increasing) system marginal cost curve

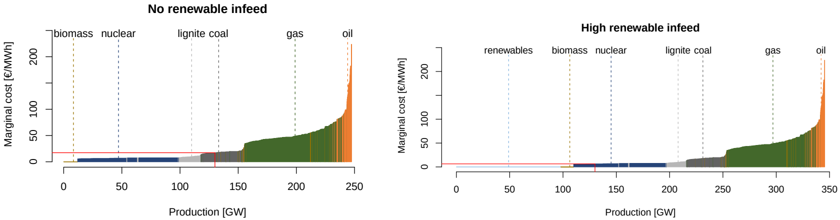


Figure: German merit order curve

How do we get aggregate cost from the merit order curve?

Aggregate and Marginal Benefit

Aggregate benefit is most beneficial way to consume Q MW of power among a *collection* of consumers

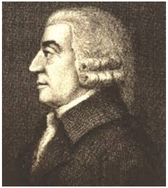
$$
\begin{aligned}
& TB_L(Q) = \max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx \\
& \text{s.t. } \sum_{l \in L} d_l = Q \\
& d_l \in \text{dom } MB_l
\end{aligned}
$$

What is the graphical analogy of the previous slide?

Marginal benefit: $MB_L(Q) = TB_L'(Q)$

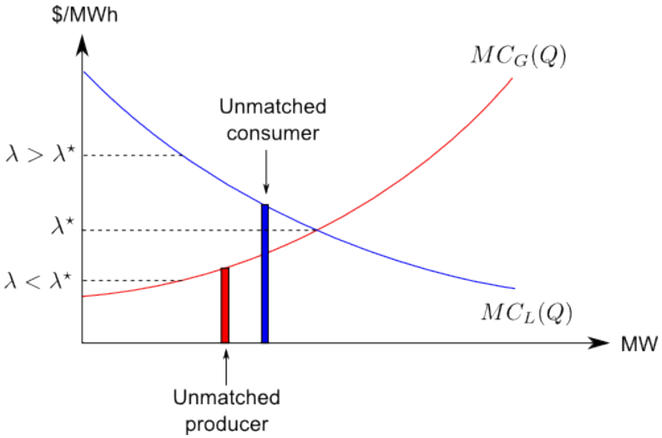
Price and Quantity Adjustment

Mechanical system dynamics are governed by Newton's laws of motion



Price adjustment and **quantity adjustment** are the 'laws of motion' for electricity markets

Price Adjustment - Graphical Illustration



Any price different from $\lambda^\star$ creates opportunities for profitable trade

Price Adjustment - Mathematical Ddescription

When demand exceeds supply, upward pressure on *prices*

When supply exceeds demand, downward pressure on *prices*

Market clearing condition

$$0 \leq \sum_{g \in G} p_g - \sum_{l \in L} d_l \perp \lambda \geq 0$$

Quantity Adjustment

- Price-taking supplier will increase *quantity* produced if marginal cost $\leq$ price, decrease output otherwise:

$$\max \lambda \cdot p_g - \int_0^{p_g} MC_g(x)dx \tag{1}$$

$$(\mu_g): \quad p_g \leq P_g \tag{2}$$

$$p_g \geq 0 \tag{3}$$

- Price-taking consumer will decrease *quantity* consumed if marginal benefit $\leq$ price, increase consumption otherwise:

$$\max \int_0^{d_l} MB_l(x)dx - \lambda \cdot d_l \tag{4}$$

$$(\mu_l): \quad d_l \leq D_l \tag{5}$$

$$d_l \geq 0 \tag{6}$$

Equilibrium, Market Clearing Price, Competitive Equilibrium, Competitive Price

- A market is in **equilibrium** when no profitable opportunities for trade exist
- The **market clearing price** is the price of a market in equilibrium
- An equilibrium in a competitive market is called a **competitive equilibrium**
- The price of a competitive market is the **competitive price**

Competitive Markets Are Efficient

The competitive equilibrium results in an allocation which is optimal for the economic dispatch problem.

Proof: Collect KKT conditions of quantity adjustment and market clearing condition of price adjustment:

$$
\begin{aligned}
\text{Suppliers: } 0 \leq p_g \quad &\perp \quad -\lambda + \mu_g + MC_g(p_g) \geq 0 \\
0 \leq \mu_g \quad &\perp \quad P_g - p_g \geq 0 \\
\text{Consumers: } 0 \leq d_l \quad &\perp \quad \lambda + \nu_l - MB_l(d_l) \geq 0 \\
0 \leq \nu_l \quad &\perp \quad D_l - d_l \geq 0 \\
\text{Market Clearing: } 0 \leq \lambda \quad &\perp \quad \sum_{g \in G} p_g - \sum_{l \in L} d_l \geq 0
\end{aligned}
$$

Identical to KKT conditions of economic dispatch

Producer and Consumer Surplus, Welfare, Efficiency

Suppose price is λ:

- **Producer surplus/profit**: profit of producers who are willing to sell

$$\lambda q_G(\lambda) - \int_0^{q_G(\lambda)} MC_G(x)dx,$$

where $q_G(\lambda)$ is quantity sold at price λ

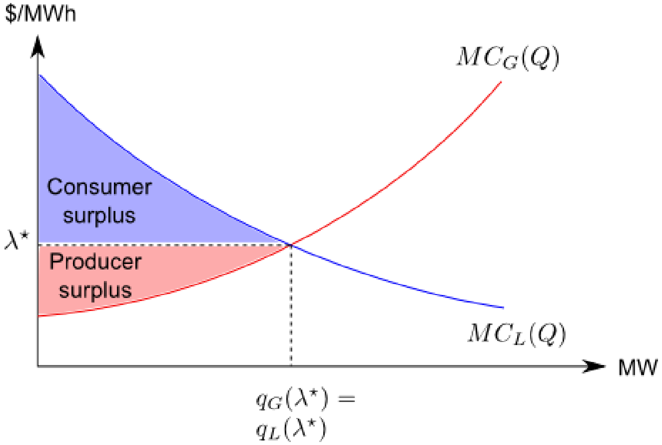
- **Consumer surplus/profit**: profit of consumers who are willing to buy

$$\int_0^{q_L(\lambda)} MB_L(x)dx - \lambda q_L(\lambda),$$

where $q_L(\lambda)$ is quantity bought at price λ

- **Welfare**: sum of producer and consumer surplus

Graphical Illustration of Producer Surplus, Consumer Surplus, Welfare



Outline

Separable Optimization

Consider the following problem:

$$
\begin{aligned}
\text{(Sep):} \quad & \max_x \sum_{i=1}^{n} f_i(x_i) \\
(\rho_i): \quad & g_i(x_i) \leq 0 \\
(\lambda): \quad & \sum_{i=1}^{n} h_i(x_i) \leq 0
\end{aligned}
$$

- $x_i \in \mathbb{R}^{n_i}$: private actions
- $f_i : \mathbb{R}^{n_i} \to \mathbb{R}$: *concave* differentiable
- $g_i : \mathbb{R}^{n_i} \to \mathbb{R}^{a_i}$ and $h_i : \mathbb{R}^{n_i} \to \mathbb{R}^{m}$: convex differentiable

Interpretation:

- m limited resources/**commodities**, n agents
- Each agent decides x_i, uses $h_i(x_i)$ of each of m resources
- For each resource: total consumption $\leq$ total production

KKT Conditions

Denote

- $\nabla_{x_i} f_i(x_i) \in \mathbb{R}^{n_i}$: gradient of f_i
- $\nabla_{x_i} g_i(x_i) \in \mathbb{R}^{a_i} \times \mathbb{R}^{n_i}$: Jacobian matrix of $g_i(x_i)$ (likewise for $\nabla_{x_i} h_i(x_i)$)

KKT conditions of (Sep):

$$-\nabla_{x_i} f_i(x_i) + (\nabla_{x_i} g_i(x_i))^T \rho_i - (\nabla_{x_i} h_i(x_i))^T \lambda = 0 \qquad (7)$$

$$0 \leq \rho_i \perp -g_i(x_i) \geq 0 \qquad (8)$$

$$0 \leq \lambda \perp -\sum_{i=1}^{n} h_i(x_i) \geq 0 \qquad (9)$$

Market for Multiple Commodities

Consider a competitive market for the m resources:

- producers are paid λ_j for selling commodity j
- consumers pay λ_j for buying commodity j
- each agent accepts price vector $\lambda^\star$ as *given* (not influenced by private decisions)

Denote q_i as vector of resources procured (or sold, if negative) by agent i, then each agent solves

$$\text{(Profit-i)} : \max_{x_i, q_i} (f_i(x_i) - (\lambda^\star)^T q_i)$$

$$(\rho_i) : \quad g_i(x_i) \leq 0,$$

$$(\lambda_i) : \quad h_i(x_i) = q_i,$$

Competitive equilibrium (for multiple products): combination of prices $\lambda^\star$, agent decisions $x_i^\star$, commodity procurements $q_i^\star$ such that:

- $(x_i^\star, q_i^\star)$ solve (Profit-i) *given* $\lambda^\star$, and
- market clearing holds:

$$0 \leq \lambda^\star \perp \sum_{i=1}^{n} q_i^\star \leq 0$$

Modeling Competitive Market Equilibrium via Optimization

Suppose KKT conditions are necessary and sufficient for the optimality of (Sep) and (Profit-i):

1. a competitive market equilibrium results in an optimal solution of (Sep), and
2. a primal-dual solution to the KKT conditions of (Sep) is a competitive equilibrium

Proof: Necessary and sufficient KKT conditions of (Profit-i):

$$
\begin{aligned}
&-\nabla_{x_i} f_i(x_i) + (\nabla_{x_i} g_i(x_i))^T \rho_i - (\nabla_{x_i} h_i(x_i))^T \lambda = 0 \\
&\lambda^\star - \lambda = 0 \\
&0 \leq \rho_i \perp -g_i(x_i) \geq 0 \\
&h_i(x_i) = q_i
\end{aligned}
$$

Proceed by comparing KKT conditions of

- (Profit-i) for all i + market clearing condition
- (Sep)

Example: Two-Firm Competitive Market

Consider the following market:

- linear marginal benefit function, $MB(Q) = a - b \cdot Q$
- Two agents, each with variable cost function TC_i

Competitive market equilibrium obtained by solving:

$$\max a \cdot d - 0.5 \cdot b \cdot d^2 - TC_1(p_1) - TC_2(p_2)$$
$$p_1 + p_2 = d$$
$$p_1, p_2, d \geq 0$$

If $p_1, p_2 > 0$, then

$$MC_1(p_1) = MC_2(p_2) = a - b \cdot (p_1 + p_2) \Leftrightarrow p_i = \frac{1}{b}(a - MC_i(p_i))$$

Example: Cournot Duopoly

Suppose agent i realizes that it influences price, solves:

$$\max (a - b \cdot (p_1 + p_2)) \cdot p_i - TC_i(p_i)$$
$$p_i \geq 0$$

Denote p_{-i} as the decision of the competing agent, if $p_i > 0$ then

$$p_i = \frac{1}{b}(a - MC_i(p_i)) - 2 \cdot p_{-i}$$

Compare with solution of previous slide, what do you observe?

Market Power

Market power: the strategic withholding of production from electricity markets by producers with the intention of *profitably* increasing prices

- Real problem in electricity markets
- Regulatory interventions (bid mitigation, price caps) can be used for mitigating market power ...
- ... but these interventions may create new problems (for example, the missing money problem)
- Strategic behavior of market agents typically analyzed using game theory (not optimization models)